Formulas of a first order language L.

Suppose L is a first order language.

An <u>atomic</u> <u>L-formula</u> is defined as follows:

- if s and t are L-terms, then $s = t$ is an atomic L-formula,

- if $t_1, \dots, t_n$ are L-terms and R is an n-ary relation symbol, then $R t_1 t_2 \dots t_n$ is an atomic L-formula.

<u>Example</u> Consider the language of set theory $L = \{\in\}$ having just one binary relation symbol. As is common, we shall allow ourselves to write $t \in s$ rather than $\in t s$, when t and s are terms of L. Now, the only L-terms are variables $x_0, x_1, x_2, \dots$, so atomic L-formulas are just $x_i \in x_j$ for $i, j \in \mathbb{N}$.

<u>Example</u> Consider the language of "ordered arithmetic": $L = \{0, 1, +, \cdot, <\}$.

then $s = (x_0 + 1) \cdot (x_7 + 1)$ and $t = 0$ are L-terms, so

$$0 < (x_0 + 1) \cdot (x_7 + 1)$$

is an atomic L-formula.

<u>Definition</u> The set of L-formulas is the smallest set such that

- any atomic formula is an L-formula,
- if A and B are L-formulas, then so are $\neg A$, $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$, $(A \leftrightarrow B)$.
- if A is an L-formula and v is a variable, then $\exists v A$ and $\forall v A$ are L-formulas.